ENTRANCE EXAMINATION FOR ADMISSION, MAY 2012.

M.Sc. Five Year Integrated Programme
(MATHEMATICS, COMPUTER SCIENCE AND STATISTICS)

COURSE CODE : 384

Register Number :

Signature of the Invigilator
(with date)

COURSE CODE : 384

Time : 2 Hours | Max : 400 Marks

Instructions to Candidates :

1. Write your Register Number within the box provided on the top of this page and fill in the page 1 of the answer sheet using pen.
2. Do not write your name anywhere in this booklet or answer sheet. Violation of this entails disqualification.
3. Read each of the question carefully and shade the relevant answer (A) or (B) or (C) or (D) in the relevant box of the ANSWER SHEET using HB pencil.
4. Avoid blind guessing. A wrong answer will fetch you –1 mark and the correct answer will fetch 4 marks.
5. Do not write anything in the question paper. Use the white sheets attached at the end for rough works.
6. Do not open the question paper until the start signal is given.
7. Do not attempt to answer after stop signal is given. Any such attempt will disqualify your candidature.
8. On stop signal, keep the question paper and the answer sheet on your table and wait for the invigilator to collect them.
9. Use of Calculators, Tables, etc. are prohibited.

1. $A = \begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$, then $(AB)^{-1}$ is

(A) $\begin{bmatrix} 28 & 67 \\ -7 & -55 \end{bmatrix}$

(B) $\begin{bmatrix} -55 & 67 \\ 23 & -28 \end{bmatrix}$

(C) $\begin{bmatrix} -67 & 23 \\ -28 & 55 \end{bmatrix}$

(D) $\begin{bmatrix} 55 & -23 \\ 7 & -4 \end{bmatrix}$

2. What is the rank of the following matrix ?

$$\begin{bmatrix} 3 & 5 & 0 & 1 \\ 2 & 6 & 3 & 2 \\ 7 & 17 & 6 & 5 \\ 11 & 21 & 3 & 5 \end{bmatrix}$$

(A) 4 (B) 3 (C) 2 (D) 1

3. The system of equations $x + 2y + 3z = 6;\ x + y + z = 3;\ 2x + 3y + 4z = 10$ has

(A) no solution

(B) exactly one solution

(C) exactly three solutions

(D) infinitely many solutions

4. If A is a square matrix of order 3 and k is a natural number, then det (kA) is

(A) $\frac{1}{k}$ det (A) (B) k det (A) (C) k^2 det (A) (D) k^3 det (A)

5. The point of contact of the line $2y = 4x + a$ with the parabola $y^2 = 4ax$ is

(A) (a, a) (B) $(a, \frac{a}{4})$ (C) $(\frac{a}{4}, \frac{a}{4})$ (D) $(\frac{a}{4}, a)$

6. The axes of the ellipse $6x^2 + 9y^2 + 12x - 36y - 12 = 0$ have lengths

(A) $6, 2\sqrt{6}$ (B) 12, 6 (C) $3, \sqrt{6}$ (D) $12, 3\sqrt{6}$

7. The angle between the asymptotes to the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ is

(A) $\pi - 2\tan^{-1}\frac{3}{4}$ (B) $\pi - 2\tan^{-1}\frac{4}{3}$ (C) $2\tan^{-1}\frac{3}{4}$ (D) $2\tan^{-1}\frac{4}{3}$

8. The sum of the squares of the y-intercepts of the tangents to the hyperbola $x^2 - 3y^2 = 24$ which make an angle of 60^0 with the x-axis is

(A) 3 (B) 6 (C) 10 (D) 20

9. Tangents to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ make angles θ_1 and θ_2 with the major axis. If $\theta_1+\theta_2=90^0$, then the locus of their point of intersection is

(A) $x^2+y^2=a^2+b^2$ (B) $x^2-y^2=a^2-b^2$

(C) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ (D) $x^2+y^2=a^2-b^2$

10. A tangent is drawn to a rectangular hyperbola. The ratio in which the portion of the tangent between the asymptotes is divided at the point of contact is

(A) 1: √2 (B) 1:√3 (C) 1: 1 (D) √2 : √3

11. The length of the latus rectum of the parabola whose vertex is (3, –1) and the directrix $y+4=0$ is

(A) 3 (B) 4 (C) 6 (D) 12

12. The distance between the foci of the ellipse $16x^2+9y^2+32x-36y=92$ is

(A) $2\sqrt{7}$ (B) $\sqrt{7}$ (C) $\frac{1}{\sqrt{7}}$ (D) $\frac{2}{\sqrt{7}}$

13. The normal to $y^2 = 4ax$ at P meets the curve again at Q. If PQ and the normal at Q

make angles θ and $\emptyset$ respectively with the axis, then $tan^2\,\theta+\tan\theta.\tan\emptyset$ is

(A) 2 (B) –2 (C) 1 (D) –1

14. An ellipse has eccentricity e. The normal at an extremity of a latus rectum passes

through an extremity of the minor axis. Then e^4+e^2 is

(A) 0 (B) 1 (C) 20 (D) $\frac{5}{16}$

15. An open – top box is to be made by uniting small congruent squares from the corners of a $12''\times12''$ sheet of tin and bending up the sides. How large should be the sides of the squares that are cut from the corners in order to make the box hold as much as possible?

(A) 4″ (B) 2.5″ (C) 3″ (D) 2″

16. The order of the element 3 in the additive group Z_8 is ...

(A) 2 (B) 4 (C) 6 (D) 8

17. Let $p = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$ be a permutation. Then the inverse of p is

(A) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix}$ (D) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}$

18. If the entries of a 2×2 matrix A are defined by the formula $a_{ij} = i^2 + j^2$, then A is ...

(A) a diagonal matrix (B) the identity matrix

(C) a symmetric matrix (D) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

19. If $y^x = x^{\sin y}$, then $\frac{dy}{dx}$ is

(A) $\frac{\sin y}{x \cos y}$ (B) $\frac{\log y - \frac{\sin y}{x}}{\cos y \log x - \frac{x}{y}}$ (C) $x \cot y$ (D) $\frac{\sin y - \log y}{\cos y - \log x}$

20. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots}}}$ ..... to ∞, then $\frac{dy}{dx}$ is

(A) $y\sqrt{\cos x}$ (B) y cos x (C) $\frac{\cos x}{2y-1}$ (D) $\frac{1}{y}$ cos x

21. If x is real, then $\lim_{x \to 0} \frac{3 \sin x - \sin 3x}{x - \sin x}$ is

(A) 0 (B) π (C) 1 (D) 24

22. What is the property of any binary operation defined on a singleton set?

(A) It forms a commutative group

(B) It is commutative but not associative

(C) It is associative but not commutative

(D) It is neither commutative nor associative

23. If $y = \tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$, then $\frac{dy}{dx}$ is

(A) x (B) $\sqrt{x}$ (C) $\frac{\pi}{2}$ (D) $\frac{1}{2}$

24. The product of the 5th roots of $1+i$ is

(A) 1 (B) i (C) $1-i$ (D) $1+i$

25. Which one of the following is a group?

(A) The set of all odd integers under addition

(B) The set of all odd integers under subtraction

(C) The set of all even integers under addition

(D) The set of all even integers under subtraction

26. The function $2x^3 - 15x^2 + 36x + 4$ has

(A) a maximum at $x = 2$ and a minimum at $x = 3$

(B) a maximum at $x = 3$ and a minimum at $x = 2$

(C) maxima at $x = 2$ and $x = 3$

(D) minima at $x = 2$ and $x = 3$

27. In the group (Q, +) the inverse of 0 is...

(A) -1 (B) 0 (C) 1 (D) ∞

28. If $y = \sin^{-1}\frac{2x}{1+x^2}$, then $\frac{dy}{dx}$ is

(A) x (B) $\frac{x}{1+x^2}$ (C) $\frac{2}{1+x^2}$ (D) $\sqrt{1-x^2}$

29. If $y = \tan^{-1}\frac{\cos x}{1+\sin x}$, then $\frac{dy}{dx}$ is

(A) $-\frac{1}{2}$ (B) $\frac{\pi}{2}$ (C) $\tan x$ (D) $\sec^2 x$

30. If $x^y = y^x$, then $\frac{dy}{dx}$ is

(A) $\frac{\log y}{\log x}$ (B) $\frac{y}{x}$ (C) $\frac{y\,(x\log y - y)}{x\,(y\log x - x)}$ (D) $\frac{y\log y - 1}{x\log x - 1}$

31. If $x+iy = \frac{3}{2+\cos\vartheta + i\sin\vartheta}$, then $4x - 3$ is

(A) $3x^2$ (B) $4x^2 - y^2$ (C) $x^2 + y^2$ (D) $\frac{y^2}{x^2}$

32. The number of real solutions of the equation $x^2 - 3\,|x| + 2 = 0$ is

(A) 2 (B) 4 (C) 1 (D) 3

33. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then

(A) $\alpha = a^2 + b^2, \beta = ab$

(B) $\alpha = a^2 + b^2, \beta = 2ab$

(C) $\alpha = a^2 + b^2, \beta = a^2 - b^2$

(D) $\alpha = 2ab, \beta = a^2 + b^2$

34. The real number x when added to its inverse gives the minimum value of the sum at x equal to

(A) 2 (B) 1 (C) – 1 (D) – 2

35. If the normal at the point $(at_1^2, 2at_1)$ on a parabola meets the parabola again in the point $(at_2^2, 2at_2)$, then

(A) $t_2 = -t_1 - \frac{2}{t_1}$

(B) $t_2 = -t_1 + \frac{2}{t_1}$

(C) $t_2 = t_1 - \frac{2}{t_1}$

(D) $t_2 = t_1 + \frac{2}{t_1}$

36. If $\vec{a}, \vec{b}, \vec{c}$ are 3 vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$, then $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is equal to

(A) 0 (B) – 25 (C) 12 (D) 60

37. If $|z^2 - 1| = |z|^2 + 1$, then z lies on

(A) the real axis

(B) an ellipse

(C) a circle

(D) the imaginary axis

38. If $(1 - p)$ is a root of quadratic equation $x^2 + px + (1 - p) = 0$, then its roots are

(A) 0, 1 (B) –1, 2 (C) 0, –1 (D) –1, 1

39. A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retires 40 meter away from the tree the angle of elevation becomes 30°. The breadth of the river is

(A) 20 m (B) 30 m (C) 40 m (D) 60 m

40. The intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB. The equation of the circle on AB as a diameter is

(A) $x^2 + y^2 - x - y = 0$

(B) $x^2 + y^2 - x + y = 0$

(C) $x^2 + y^2 + x + y = 0$

(D) $x^2 + y^2 + x - y = 0$

41. For what value of m the vectors $\vec{a} = 5\vec{i} - 9\vec{j} + 2\vec{k}$ and $\vec{b} = m\vec{i} + 2\vec{j} + \vec{k}$ are perpendicular to each other?

(A) 4 (B) $\frac{16}{5}$ (C) 0 (D) $3\sqrt{3}$

42. The area of the parallelogram with diagonals $3\vec{i} + \vec{j} - 2\vec{k}$ and $\vec{i} - 3\vec{j} + 4\vec{k}$ is

(A) $8\sqrt{3}$ (B) $14\sqrt{3}$ (C) $5\sqrt{3}$ (D) $3\sqrt{3}$

43. If $\vec{a}$ and $\vec{b}$ are unit vectors inclined at an angle θ, then $\dfrac{\sin\frac{\theta}{2}}{\left|\vec{a} - \vec{b}\right|}$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1

44. For any three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$, what is $\left[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}\right]$?

(A) 0 (B) $[\bar{a}, \bar{b}, \bar{c}]$ (C) $2[\bar{a}, \bar{b}, \bar{c}]$ (D) $3[\bar{a}, \bar{b}, \bar{c}]$

45. If the volume of a parallelopiped whose edges are represented by $-12\vec{i} + \lambda\vec{j}$, $3\vec{j} - \vec{k}$, $2\vec{i} + \vec{j} - 15\vec{k}$ is 546, what is the value of λ?

(A) -3 (B) -6 (C) -2 (D) $-\sqrt{2}$

46. If the work done by a force $\vec{F} = a\vec{i} + \vec{j} + \vec{k}$ in moving the point of application from (1, 1, 1) to (2, 2, 2) along a straight line is found to be 4 units, then the value of a is

(A) 1 (B) 2 (C) 3 (D) 4

47. If θ is the angle between the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-4}{6}$ and $x - 1 = = \frac{y+2}{2} = \frac{z-4}{2}$, then what is $\cos\theta$?

(A) $\frac{1}{21}$ (B) $\frac{\sqrt{2}}{21}$ (C) $\frac{20}{21}$ (D) $\frac{5\sqrt{2}}{21}$

48. What is the shortest distance between the skew lines $\vec{r} = (\vec{i} - \vec{j}) + (2\vec{i} + \vec{j} + \vec{k})$ and $\vec{r} = (\vec{i} + \vec{j} - \vec{k}) + \mu(2\vec{i} - \vec{j} - \vec{k})$?

(A) $\frac{3}{\sqrt{2}}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{2}{\sqrt{3}}$ (D) $\frac{\sqrt{2}}{3}$

49. What is the value of λ if the points (3, 2, –4), (9, 8, –10) and (λ, 4, –6) are collinear?

(A) 0 (B) 1 (C) –4 (D) 5

50. What is the distance from the point (1, –1, 2) to the plane $\vec{r} = (\vec{i} + \vec{j} + \vec{k}) + s(\vec{i} - \vec{j}) + t(\vec{j} - \vec{k})$?

(A) $\frac{1}{\sqrt{3}}$ (B) $3\sqrt{3}$ (C) $\frac{5}{\sqrt{3}}$ (D) $8\sqrt{3}$

51. What is the value of λ if the planes $\vec{r}.(2\vec{i} + \lambda\vec{j} - 3\vec{k}) = 10$ and $\vec{r}.(\lambda\vec{i} + 3\vec{j} + \vec{k}) = 5$ are perpendicular?

(A) 5 (B) 15 (C) $\frac{1}{5}$ (D) $\frac{3}{5}$

52. If $[\vec{a} \times \vec{b},\ \vec{b} \times \vec{c},\ \vec{c} \times \vec{a} = 16]$, what is $[\vec{a},\ \vec{b},\ \vec{c}]$?

(A) 0 (B) 4 (C) 8 (D) 32

53. Which one of the following is true?

(A) $p \vee (\sim p)$ is a tautology and $p \wedge (\sim p)$ is a contradiction

(B) $p \vee (\sim p)$ is a contradiction and $p \wedge (\sim p)$ is a tautology

(C) Each one of $p \vee (\sim p)$ and $p \wedge (\sim p)$ is a tautology

(D) Each one of $p \vee (\sim p)$ and $p \wedge (\sim p)$ is a contradiction

54. The value of $\left(\frac{-1 + i\sqrt{3}}{2}\right)^{30} + \left(\frac{-1 - i\sqrt{3}}{2}\right)^{30}$ is

(A) 0 (B) 1 (C) –1 (D) 2

55. $\frac{1 + e^{i\theta}}{1 + e^{-i\theta}}$ is equal to

(A) $\cos\theta + i\sin\theta$ (B) $\cos\theta - i\sin\theta$ (C) $\sin\theta - i\cos\theta$ (D) $\sin\theta + i\cos\theta$

56. The conjugate of $i^{21}+i^{22}+i^{23}+i^{24}$ is

(A) 0 (B) 1 (C) i (D) $-i$

57. If ω is a cube root of unity, then $(1-\omega+\omega^2)^4 + (1+\omega-\omega^2)^4$ equal to

(A) –4 (B) –8 (C) –16 (D) –32

58. The modulus of the complex number $\frac{(1+3i)(1-2i)}{(3+4i)}$ is

(A) 1 (B) $\frac{1}{\sqrt{2}}$ (C) $2\sqrt{2}$ (D) $\sqrt{2}$

59. If $\bar{z}$ lies in the second quadrant, then z lies in the

(A) first quadrant (B) second quadrant

(C) third quadrant (D) fourth quadrant

60. If p represents the variable complex number z, what is the locus of p if $\arg\left(\frac{z-1}{z+1}\right)=\left(\frac{\pi}{3}\right)$?

(A) straight line (B) parabola (C) ellipse (D) circle

61. The points representing the complex numbers $2i, 1+i, 4+4i$ *and* $3+5i$ $\frac{x^2}{e^x}$ on the argand plane form a

(A) straight line (B) rectangle (C) square (D) rhombus

62. If θ is the angle between the curves $y=x^2$ and $y=(x-2)^2$ at (1, 1), then the value of $\tan\theta$ is

(A) $\frac{1}{3}$ (B) $\sqrt{3}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{4}{3}$

63. What is the value of k^2 in order that the curves $y^2=x$ and $xy=k$ are orthogonal ?

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{1}{16}$

64. What is the limit of $\frac{x^2}{e^x}$ as x tends to infinity?

(A) 0 (B) 1 (C) e (D) e^x

65. The maximum and minimum values of the function $f(x)=x^3-12x+1$ *in* (–3, 5) are

(A) 66, 10 (B) 66, –10 (C) 66, –15 (D) 10, 66

66. What is the point on the parabola $y^2 = 2x$ that is closest to the point (4, 3) ?

(A) The origin (B) (2, 2) (C) (4, 2) (D) $(1, \sqrt{2})$

67. If $u = \log(\tan x + \tan y + \tan z)$, then $\sum \sin 2x \dfrac{\partial u}{\partial x}$ is equal to

(A) 0 (B) 1 (C) 2 (D) –1

68. If $u = (x-y)(y-z)(z-x)$, then $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z}$ is equal to

(A) 0 (B) $x+y+z$ (C) $-(x+y+z)$ (D) xyz

69. $\int_0^1 \dfrac{dx}{\sqrt{4-x^2}}$ is equal to

(A) 0 (B) $\dfrac{\pi}{2}$ (C) $\dfrac{\pi}{3}$ (D) $\dfrac{\pi}{6}$

70. $\int_0^{\frac{\pi}{2}} \dfrac{\sin x}{1+\cos^2 x} dx$ is equal to

(A) 0 (B) $\dfrac{\pi}{2}$ (C) $\dfrac{\pi}{4}$ (D) π

71. $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^2 x \, dx$ is equal to

(A) $\dfrac{31}{8}$ (B) $\dfrac{3\pi}{8}$ (C) $\dfrac{\pi}{16}$ (D) $\dfrac{\pi}{32}$

72. The area of the region bounded by the lines $x = y + 3$, $x = 1$ and $x = 5$ is

(A) 1 square unit (B) 2 square units

(C) 3 square units (D) 4 square units

73. The area between the curves $y = \sin x$ and $y = \cos x$ and the lines $x = 0$ and $x = \pi$ is

(A) 1 square unit (B) $\sqrt{2}$ square units

(C) $\frac{1}{\sqrt{2}}$ square unit (D) $2\sqrt{2}$ square units

74. The common area enclosed by the parabolas $4y^2 = 9x$ and $3x^2 = 16y$ is

(A) 1 square unit (B) 2 square units

(C) 3 square units (D) 4 square units

75. The order and degree of the differential equation $\frac{d^2y}{dx^2} = \left(y - \left(\frac{dy}{dx}\right)^3\right)^{\frac{2}{3}}$ are respectively

(A) 1, 1 (B) 2, 2 (C) 2, 3 (D) 3, 3

76. The integrating factor for the differential equation (x + 1) $\frac{dy}{dx} - y = e^x (x+1)^2$ is

(A) $\frac{1}{x+1}$ (B) $x+1$ (C) $\frac{x}{x+1}$ (D) $e^x \; x+1$

77. In the set of integers with the operation * defined by a * b = a + b – a b, what is the value of 2 * (4 * 6) ?

(A) 16 (B) 12 (C) –22 (D) 48

78. If A and B are $m \times n$ matrices, then the size of $A^T + B^T$ is

(A) $m \times n$ (B) $m \times m$ (C) $n \times n$ (D) $n \times m$

79. The probability mass function of a random variable X is given below:

x	0	1	2	3
$p(x)$	c	$3c$	$5c$	$7c$

The value of c is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{1}{16}$

80. If X and Y are two random variables having the joint density function

$f(x,y) = \frac{1}{8}(6 - x - y); 0 < x < 2,\ 2 < y < 4,$

then the value of $P(X < 1/Y < 3)$ is

(A) $\frac{1}{5}$ (B) $\frac{1}{3}$ (C) $\frac{3}{5}$ (D) $\frac{2}{3}$

81. A random variable X has probability function as follows:

x	-1	0	1
$p(x)$	$3c^2$	$4c - 10c^2$	$5c - 1$

Then (i) $E(3X+1)$ and (ii) $E(X^2)$ are

(A) 1.8; 05 (B) 1.4; 0.6 (C) 1.2; 0.7 (D) 1.9; 0.7

82. A box contains 2^n ticket among which n_{C_r} tickets bear the number r ($r = 0,1,2,\ldots,n$). A group of m tickets is drawn. What is the expectation of the sum of their numbers?

(A) $\frac{m}{2n}$ (B) $\frac{m}{n}$ (C) $\frac{2m}{n}$ (D) $\frac{mn}{2}$

83. If a and b are constants, then $V(a^2X + b)$ is

(A) $a^2V(X)$ (B) $a^2V(X) + V(b)$

(C) $a^4V(X)$ (D) $V(X) + v(a^2b)$

84. A continuous random variable X has a pdf given by

$$f(x) = \begin{cases} kxe^{-\lambda x}; & x \geq 0, \lambda > 0 \\ 0 & \text{otherwise} \end{cases}$$

Then the constant k ; mean and variance of X are

(A) $\lambda\ ; \frac{6}{\lambda^2}, \frac{2}{\lambda^2}$ (B) $\lambda^2\ ; \frac{6}{\lambda^2}, \frac{2}{\lambda^2}$

(C) $\lambda^2\ ; \frac{6}{\lambda}, \frac{2}{\lambda^2}$ (D) $\lambda^2\ ; \frac{6}{\lambda^2}, \frac{2}{\lambda}$

85. For a variable X, $E(X) = 10$ and $\mathrm{Var}(X) = 25$. What are the positive values of "a" and "b" such that $Y = aX–B$ has expectation zero and variance 1?

(A) $\frac{1}{5}, 2$ (B) $\frac{1}{2}, 5$ (C) $5, 2$ (D) $2, 5$

86. What value of k guarantees that $P(|X - 7| \leq k) \geq 0.99$ assuming $\mu = 7, \sigma = 2$?

(A) $k = 20$ (B) $k = 15$ (C) $k = 10$ (D) $k = 5$

87. The mean and standard deviation of a binomial distribution are 5 and 2 respectively. What is the probability mass function of binomial distribution?

(A) $p(x) = 25_{C_x} \left(\frac{1}{5}\right)^x \left(\frac{4}{5}\right)^{25}$, $x = 0,1,2, \ldots\ldots,25$

(B) $p(x) = \left(\frac{1}{5}\right)^x \left(\frac{4}{5}\right)^{25-x}$, $x = 0,1,2, \ldots\ldots,25$

(C) $p(x) = 25_{C_x} \left(\frac{1}{5}\right)^x \left(\frac{4}{5}\right)^{25-x}$, $x = 1,2,3, \ldots\ldots,25$

(D) $p(x) = 25_{C_x} \left(\frac{1}{5}\right)^{25} \left(\frac{4}{5}\right)^{25-x}$, $x = 0,1,2, \ldots\ldots,25$

88. The mean and variance of a binomial distribution are 8 and 6 respectively. Then $P(X \geq 2)$ is

(A) $1 - \frac{35}{4}\left(\frac{1}{4}\right)^{31}$

(B) $1 - \frac{35}{4}\left(\frac{1}{2}\right)^{31}$

(C) $1 - \frac{35}{4}\left(\frac{3}{4}\right)^{31}$

(D) $1 - \frac{35}{3}\left(\frac{1}{4}\right)^{31}$

89. In a throw of a die, obtaining 5 or 6 is considered a success. The mean number of successes and standard deviation in eight throws of a die are

(A) $n = 5$, sd = 4/3

(B) $n = 6$, sd = 4/3

(C) $n = 8$, sd = 4/3

(D) $n = 4$, sd = 4/3

90. With usual notation, find p for a binomial random variable X if n=6 and 9P(X=4) = P(X=2).

(A) 1/4 (B) 1/2 (C) 1/5 (D) 1/3

91. Ten coins are tossed simultaneously. Find the probability of getting (i) at least 7 heads and (ii) exactly seven heads.

(A) 15/64, 11/128

(B) 11/64, 15/128

(C) 11/64, 11/128

(D) 15/64, 15/128

92. Let X be a Poisson random variable such that $P(X=1)=0.3$ and $P(X=2)=0.2$. Then $P(X=0)$ is

(A) 0.364 (B) 0.264 (C) 0.464 (D) 0.465

93. The number of accidents in a year attributed to taxi drivers in a city follows a Poisson distribution with mean equal to 3. Out of 1,000 taxi drivers, find approximately the number of drivers with (i) no accidents in a year and (ii) more than 3 accidents in a year.

(A) 0.04, 350 (B) 0.05, 350 (C) 0.05, 320 (D) 0.40, 320

94. If 3% of the electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs exactly five bulbs are defective. (exp(–3) = –.498)

(A) 0.20 (B) 0.24 (C) 0.10 (D) 0.01

95. If X_1, X_2 are two independent Poisson random variables with parameters λ_1 and λ_2, then $X_1 + X_2$ is a -------------------- random variable with parameter $\lambda_1 + \lambda_2$.

(A) binomial (B) normal (C) poisson (D) exponential

96. For a Poisson distribution, which one of the following is correct?

(A) mean is half of the variance

(B) mean and variance are the same

(C) mean > variance

(D) mean is zero and variance is one

97. The random variable X follows standard normal distribution. Find the mean and variance of X.

(A) Mean = 0, variance = 1

(B) Mean = 0, variance = 0.5

(C) Mean = 1, variance = 0

(D) Mean = 1, variance = 0.5

98. Let X be normally distributed with mean 8 and standard deviation 4. Find (i) $P(5 \leq X \leq 10)$ and (ii) $P(X \leq 5)$

(A) 0.35, 0.22 (B) 0.46, 0.22 (C) 0.54, 0.22 (D) 0.40, 0.22

99. What is the moment generating function of a normal distribution?

(A) $M_x(t) = exp\left(t\mu + \frac{\sigma^2 t^2}{2}\right)$

(B) $M_x(t) = exp\left(\mu + \frac{\sigma^2 t^2}{2}\right)$

(C) $M_x(t) = exp\left(t\mu + \frac{\sigma^2 t}{2}\right)$

(D) $M_x(t) = exp\left(t\mu + \frac{\sigma t^2}{2}\right)$

100. Let X follow a normal distribution with mean μ and variance σ^2. Which one of the following statements is not true?

(A) The graph of the distribution is symmetrical about $x = \mu$

(B) The mean, median and mode coincide at $x = \mu$

(C) It has points of inflexion at $x - \mu \pm \sigma$

(D) The maximum probability is at $x = \mu$ and it is equal to $\frac{1}{\sqrt{2\pi}}$.